

# Nonabelian Confinement near Nontrivial Conformal Vacua

Kenichi KONISHI <sup>(1,2)</sup>,  
Giacomo MARMORINI <sup>(3,2)</sup>, Naoto YOKOI <sup>(2,1)</sup>

*Dipartimento di Fisica “E. Fermi” – Università di Pisa <sup>(1)</sup>,  
Istituto Nazionale di Fisica Nucleare – Sezione di Pisa <sup>(2)</sup>,  
Largo Pontecorvo, 3, Ed. C, 56127 Pisa, Italy*

*Scuola Normale Superiore - Pisa <sup>(3)</sup>, Piazza dei Cavalieri 7, Pisa, Italy*

konishi(at)df.unipi.it, g.marmorini(at)sns.it, yokoi(at)df.unipi.it

## Abstract

We discuss some aspects of confinement and dynamical symmetry breaking in the so-called nonabelian Argyres-Douglas vacua, which occur very generally in supersymmetric theories. These systems are characterized by strongly-coupled nonabelian monopoles and dyons; confinement and dynamical symmetry breaking are caused by the condensation of monopole composites, rather than by condensation of single weakly-coupled monopoles. In general, there are strong constraints on which kind of monopoles can appear as the infrared degrees of freedom, related to the proper realization of the global symmetry of the theory. Drawing analogies to some of the phenomena found here, we make a speculation on the ground state of the standard QCD.

## 1. Introduction

Our understanding of the quantum behavior of nonabelian monopoles in 4D gauge theories [1]-[6] has greatly improved in the last few years. By exploiting the knowledge of the exact solutions in theories with  $\mathcal{N} = 1$  or  $\mathcal{N} = 2$  supersymmetry [7]-[12] we know for instance that there are systems in which the 't Hooft-Mandelstam scenario of confinement (dual Meissner effect) [13] is indeed realized dynamically. However, in all such systems confinement is accompanied by dynamical abelianization, with characteristic enrichment of Regge trajectories, a feature which is not shared by quantum chromodynamics (QCD) [14, 15].

Actually, a more general class of supersymmetric theories with massless quark fields exhibit a rather different picture of confinement and dynamical symmetry breaking. First of all, confinement is typically described as a dual Meissner effect, of a *non-abelian kind*, where both condensing monopoles and confining strings carry unbroken nonabelian fluxes. The properties of these nonabelian superconductors [16, 17] are being intensely investigated [18].

These nonabelian confining vacua studied so far, can be further classified, roughly speaking, into two subclasses of systems; the first one characterized by weakly coupled monopoles, and the second involving relatively non-local set of strongly-coupled monopoles and dyons. The first class of systems can be described by a local (dual) effective Lagrangian, which can be easily and exactly analysed, for instance, by using the Seiberg-Witten solutions; confinement and dynamical symmetry breaking are explicitly described in terms of the nonabelian monopoles and vortices. From the point of view of classification of conformal theories, these correspond to trivial (infrared-free) conformal theories, perturbed by a  $\mathcal{N} = 1$  potentials. Although interesting, this first group of systems (typical examples are the “ $r$  vacua” of  $\mathcal{N} = 2$ ,  $SU(N)$  SQCD) cannot be considered as a good model for QCD, either. For instance, the problem raised in [14] of non linear Regge trajectories would persist also in these cases.

In the present paper, we concentrate our attention on the second class of non-abelian vacua, where the infrared degrees of freedom involve relatively non local and in general strongly-coupled dyons. The theory is close to (*i.e.* a perturbation of) a non-trivial fixed-point theory, which is a superconformal field theory (SCFT). These systems unfortunately defy the traditional effective-Lagrangian approach, and as a result it is much more difficult to understand what is happening in the infrared. Never-

theless, by fully mobilizing our general knowledge such as the Nambu-Goldstone theorem, Seiberg-Witten curves, instanton-induced effective action, holomorphic properties due to supersymmetry, vacuum counting, decoupling theorem, exact anomalous and non-anomalous Ward-Takahashi identities, universality of conformal theories, Seiberg’s duality, some new results on  $\mathcal{N} = 1$  susy gauge theories [11, 12], and so on, one can get a fairly precise picture of physics in these vacua. It is the purpose of this paper to discuss various aspects of these confining vacua, characterized by highly non-trivial monopole dynamics.

## 2. Physics of nonabelian Argyres-Douglas vacua: confinement versus dynamical symmetry breaking

Study of nonabelian generalization of electromagnetic duality goes back to work even before the seventies [1]-[3]. The idea that such duality is a *symmetry* of the system (Montonen and Olive [2]), is believed to hold in superconformal  $\mathcal{N} = 4$  gauge theories. In the context of asymptotically free gauge theories, the first example of nonabelian duality was found by Seiberg [4] and others [5], in the context of  $\mathcal{N} = 1$  theories. In the supersymmetric QCD (SQCD), with the number of flavor in the so-called conformal window,  $\frac{3N_c}{2} < N_f < 3N_c$ , the theory at the origin of the vacuum moduli space (with all scalar VEVs vanishing) flows into an infrared fixed point: the theory becomes superconformal (SCFT). This theory is described either by the original  $SU(N)$  QCD, or by the dual  $SU(N_f - N)$  theory with  $N_f$  dual quarks. A review on various dualities and their relations can be found in [15].

Somewhat analogous superconformal theories, appearing as an infrared fixed-point theory, were discovered by Argyres and Douglas [19] and others [20, 21, 22], in the context of  $\mathcal{N} = 2$  supersymmetric gauge theories. The example studied in [19] is a pure  $\mathcal{N} = 2$  supersymmetric  $SU(3)$  Yang-Mills theory: the infrared theory in question is an effective  $U(1) \times U(1)$  theory, where one of the  $U(1)$  factor is strongly coupled. Interest in this kind of systems (which we shall call Argyres-Douglas vacua in this paper) arises from the fact that, in contrast to the cases discussed by Seiberg and others, the nature of the low-energy degrees of freedom is better understood and, in particular, because they are known to include a mutually nonlocal set of dyon fields. In the specific case of the interacting  $U(1)$  SCFT of Argyres and Douglas, the IR “matter” degrees of freedom are a magnetic monopole, a dyon and an electron. The

beta function cancel among the contributions from these relatively nonlocal fields [19, 20]. See also [23] for a further discussion on the renormalization group flow near such fixed points.

We are interested here in nonabelian analogues of the Argyres-Douglas vacuum, appearing in various theories.

## 2.1. Argyres-Douglas vacua of $SU(N)$ supersymmetric QCD

In the  $\mathcal{N} = 2$   $SU(N)$  SQCD, the degrees of freedom in UV are (in  $\mathcal{N} = 1$  formalism) the gauge multiplet  $W = (A_\mu, \lambda)$ , the adjoint chiral multiplet  $\Phi = (\phi, \psi)$ , and the quark multiplets  $Q_i = (q, \psi_Q)_i$  and  $\tilde{Q}_i = (\tilde{q}, \tilde{\psi}_Q)_i$ , in the fundamental (antifundamental) representation of the gauge group ( $i = 1, 2, \dots, N_f$ ). We start with small quark masses  $m_i$  and we break the  $\mathcal{N} = 2$  supersymmetry to  $\mathcal{N} = 1$  by a small superpotential (mass term) for the adjoint chiral multiplet,

$$\mu \text{Tr } \Phi^2|_F = \mu \psi \psi + \dots \quad (2.1)$$

This theory has a large vacuum degeneracy (vacuum moduli): the vacuum we are interested classically corresponds to the one characterized by the scalar VEV,

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \text{diag}(-m_1, -m_2, \dots, -m_r, c, c, \dots, c), \quad c = \frac{1}{N_c - r} \sum_{k=1}^r m_k, \quad (2.2)$$

$$\langle q_i^\alpha \rangle = \delta_i^\alpha \sqrt{\mu m_i}, \quad i = 1, 2, \dots, r, \quad (2.3)$$

that is, with the gauge group classically broken to

$$SU(N_c) \rightarrow \prod_i U_i(1) \times SU(N_c - r) \xrightarrow{m_i \rightarrow m} U(r) \times SU(N_c - r). \quad (2.4)$$

The  $SU(N_c - r)$  sector becomes strongly interacting in the infrared and breaks itself to the maximally abelian subgroup [24]. The  $\prod_i U_i(1)$  factor group in Eq. (2.4), on the other hand, gets enhanced into the  $U(r)$  group in the equal mass (and/or massless) limit  $m_i \rightarrow m$  (or  $m_i \rightarrow 0$ ). This  $SU(r)$  is infrared free if  $r < \frac{N_f}{2}$ , as is seen from the effective quark mass terms

$$Q_i (\langle \sqrt{2} \Phi \rangle + m_i) \tilde{Q}_i, \quad (2.5)$$

and Eq. (2.2).

The critical cases  $r = \frac{N_f}{2}$ , where the theory becomes conformal invariant at low-energies, are our main interest in this section.

The simplest such example appears in the  $SU(3)$  gauge theory with  $N_f = 4$  flavors. In the Argyres-Douglas vacuum of this theory, the low-energy effective gauge group is  $SU(2) \times U(1)$ . If the masses  $m$  are large compared to the dynamical scale of the theory  $\Lambda$ , the theory is basically the local  $SU(2)$  theory with  $N_f = 4$  flavors, which as is well known, is conformally invariant, with  $\beta = 0$ .

As  $m \rightarrow 0$ , however, the flow of the theory into the infrared fixed point occurs in a nontrivial way. The low-energy physics of  $\mathcal{N} = 1$  vacua are encoded in the structure of the singularities of the Seiberg-Witten curve [9], which in this case reads,

$$y^2 = \prod_{a=1}^3 (x - \phi_a)^2 - \prod_{i=1}^4 (x + m_i) \equiv (x^3 - Ux - V)^2 - 4\Lambda^2 \prod_{i=1}^4 (x + m_i), \quad (2.6)$$

where  $U = \frac{1}{2}\langle \text{Tr}\Phi^2 \rangle$  and  $V = \frac{1}{3}\langle \text{Tr}\Phi^3 \rangle$  parametrize inequivalent vacua<sup>1</sup>. For *equal bare quark masses* ( $m_i = m$ ), it simplifies:

$$y^2 = \prod_{a=1}^3 (x - \phi_a)^2 - (x + m)^4 \equiv (x^3 - Ux - V)^2 - 4\Lambda^2 (x + m)^4. \quad (2.7)$$

The  $r = 2$  (Argyres-Douglas) vacuum corresponds to the point,  $\text{diag } \phi = (-m, -m, 2m)$ , *i.e.*,

$$U = U^* = 3m^2; \quad V = V^* = 2m^3, \quad (2.8)$$

where the curve exhibits a singular behavior (the bi-torus degenerates into a sphere),

$$y^2 \propto (x + m)^4 \quad (2.9)$$

corresponding to the unbroken  $SU(2)$  symmetry<sup>2</sup>.

In order to find out the nature of the infrared degrees of freedom in the SCFT vacuum (2.8) one must determine the loci in the  $(U, V)$  space near  $(U^*, V^*)$ , along

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<sup>1</sup>Eq.(2.6) represents a one dimensional complex surface (curve), which can be thought as a hypertorus with genus two. The low-energy effective coupling constant and  $\theta$  parameter, as well as the masses of the short multiplets (BPS states) are expressed as integrals of certain differential forms along the nontrivial cycles on this hypertorus, which constitutes the Seiberg-Witten solution [7]-[10]. Curves such as (2.6) compactly encode all the perturbative and nonperturbative effects.

<sup>2</sup>The singularity (2.8) splits into six separate nearby singularities when the quark masses are taken to be slightly unequal and generic (the sextet vacua).

which some massless particles are present (corresponding to a double branch point of the curve Eq. (2.7)), and study how various quantities transform as one goes around such loci (monodromy matrices). This problem has been analyzed in detail in [25]; the low-energy degrees of freedom are found to carry the magnetic and electric  $U(1) \times U(1)$  charges, shown in Table 1, with the first  $U(1)$  factor (magnetic or electric) referring to the subgroup of the  $SU(2)$ . The system having  $\mathcal{N} = 2$  supersymmetry, there are also particles  $\tilde{M}^\alpha$ ,  $\tilde{D}^\alpha$ ,  $\tilde{E}^\alpha$ , with conjugate gauge quantum numbers.

Particles	$(g_1, g_2; q_1, q_2)$
$M_1, M_2$	$(\pm 1, 1; 0, 0)^4$
$D_1, D_2$	$(\pm 2, -2; \pm 1, 0)$
$E_1, E_2$	$(0, 2; \pm 1, 0)$

Table 1: The charges of the massless doublets.  $g_i$  ( $q_i$ ) is the magnetic (electric) charge with respect to the  $i$ -th  $U(1)$  factor.

The superscript in the table indicates the multiplicity of the massless particle present. The pair of particles carrying opposite charges with respect to the first  $U(1)$  (magnetic or electric) factor, can be interpreted naturally as forming a doublet of the  $SU(2)$ . This way we arrive at the conclusion that there are massless monopole doublets carrying the 4 flavor charge of  $SU(4)$ , and a dyonic and an electric doublets which are singlets of the global  $SU(4)$ . The particles in the table carry indeed relatively nonlocal charges, *i.e.*, nonvanishing relative Dirac unit [13],

$$\sum_{i=1}^2 (g_i q'_i - g'_i q_i) \neq 0, \quad \text{Mod } [N] \quad (2.10)$$

(for  $SU(N)$ , here  $N = 3$ ), and the theory is superconformal [20]. Let us recall that the cancellation of the beta function has been checked in [25] by generalizing the argument by Argyres and Douglas [19], to our nonabelian SCFT. Here, in contrast to the case studied in [19], the gauge multiplet contributes. In the dual base in which  $M, \tilde{M}$  fields are coupled minimally to the dual gauge bosons, the contribution of four flavors of  $M, \tilde{M}$  cancel that of the  $SU(2)$  gauge multiplet. The contribution of  $D$  and  $E$  fields to the beta function must be computed in the base where these are coupled locally to the appropriate (dyonic or original) gauge bosons, then the result transformed back to the magnetic base. It turns out they cancel precisely

$$1 + (2\tau^* + 1)^2 = 0, \quad (2.11)$$

at the critical coupling constant,  $\tau^* = \frac{-1+i}{2}$ , the value found independently from the study of the shape of the hypertorus in the SCFT limit [25].

The  $\mu \Phi^2$  perturbation breaks superconformal invariance, and the theory confines.

This precise knowledge on the infrared degrees of freedom from the Seiberg-Witten curve can be combined with the pattern of the symmetry breaking following from the independent analysis made at large  $\mu$ . Due to the holomorphic dependence of physics on the parameter  $\mu$  there cannot be any phase transition at a finite value of  $|\mu|$ . At large  $\mu$ , (where the nonperturbative dynamics is that of  $\mathcal{N} = 1$  theory) the instanton-induced superpotential is known from the earlier studies, and one can easily determine the symmetry breaking pattern in this vacuum [27],

$$SU(4) \times U(1) \rightarrow U(2) \times U(2). \quad (2.12)$$

One is then led to conclude that the symmetry breaking at small  $\mu$  is caused by the bilinear condensate <sup>3</sup>

$$\langle \epsilon_{\alpha\beta} M_i^\alpha M_j^\beta \rangle \neq 0, \quad \langle \epsilon^{\alpha\beta} \tilde{M}_\alpha^i \tilde{M}_\beta^j \rangle \neq 0, \quad (2.13)$$

due to the strong magnetic  $SU(2)$  interactions. Note that the condensate (2.13) is antisymmetric in flavor and, as required, reproduces the correct symmetry breaking pattern, (2.12).

A crucial observation made in [25] is the following. As the bare quark masses  $m_i$  are taken slightly different from each other, the vacuum we are considering splits into six nearby vacua. Each of them is a *local* Abelian  $U(1) \times U(1)$  theory, with a pair of massless monopoles, which condense upon  $\mathcal{N} = 1$  perturbation. The problem is that the condensates of the abelian monopoles

$$\langle M \rangle = \text{const.} \sqrt{\mu\Lambda} \quad (2.14)$$

in each vacuum, in fact, vanish (const.  $\rightarrow 0$ ) in the limit  $m_i \rightarrow m$  we are interested in.<sup>4</sup> Dual superconductor picture of confinement à la 't Hooft-Mandelstam [13] (with  $U(1)^2$  gauge symmetry) is, therefore, not the correct mechanism of confinement in the present system.

We believe that here the confinement (and the symmetry breaking) is caused by the strong interaction effects among the nonabelian monopoles, (2.13), which are

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<sup>3</sup> $\alpha, \beta = 1, 2$  are dual color indices,  $i, j = 1, \dots, 4$  are the flavor indices of  $G_F = SU(4)$ .

<sup>4</sup>Analogous phenomenon has been noted in [26].

entirely missed in the abelian local effective-action description at each of the six vacua, formally valid before the SCFT limit is taken. Actually, the validity (in the mass scale) of the abelian local effective action of each vacuum, is limited from above by the masses of the massive nonabelian gauge bosons, which tend to zero in the  $m_i \rightarrow m$  limit. In order to correctly understand the infrared physics, one must take into account the full  $SU(2)$  gauge interactions.

In more general  $r = \frac{N_f}{2}$  vacua of  $SU(N)$  theory, one expects again that nonabelian monopoles  $M_i^\alpha$  in the fundamental representation of  $SU(\frac{N_f}{2})$  gauge group and in the fundamental representation of the global  $SU(N_f)$  group, will be responsible for confinement/dynamical symmetry breaking. From the known symmetry breaking pattern [27]

$$SU(N_f) \times U(1) \rightarrow U(\frac{N_f}{2}) \times U(\frac{N_f}{2}), \quad (2.15)$$

we conclude that the baryonlike condensate

$$\langle \epsilon_{\alpha_1 \alpha_2 \dots \alpha_{N_f/2}} M_{i_1}^{\alpha_1} M_{i_2}^{\alpha_2} \dots M_{i_{N_f/2}}^{\alpha_{N_f/2}} \rangle \neq 0, \quad (2.16)$$

forms in this vacuum. The mesonlike condensates

$$\langle M_i^\alpha \tilde{M}_\alpha^j \rangle = \text{const. } \delta_i^j, \quad (2.17)$$

might also form, but they do not modify the symmetry breaking pattern (2.16), being singlet of  $G_F = SU(N_f) \times U(1)$ .

### Baryon number conservation

There is, actually, one subtle issue in Eq.(2.13) and Eq.(2.16) concerning the quark (baryon) number conservation. The analysis made at large  $\mu$  (where the order parameter of symmetry breaking is the meson condensates  $\sim \langle Q \tilde{Q} \rangle$  only) shows that the baryon number  $U(1)$  is unbroken, while the nonabelian part  $SU(N_f)$  is broken, as in Eq.(2.12) and Eq.(2.15). On the other hand, the infrared degrees of freedom of the theory at small  $\mu$  are the nonabelian monopoles, carrying in general nontrivial flavor quantum numbers, as 4 of  $M, \tilde{M}$  of Table 1. Since the soliton monopoles usually carry also fractional  $U(1)$  charges - unless forbidden by a symmetry - one might wonder whether our picture of the infrared physics based on strongly interacting monopoles is consistent.

It is a highly nontrivial check of consistency of our claim, that the dual quarks of the quantum  $r$  vacua ( $r \leq \frac{N_f}{2}$ ) carry exactly vanishing baryon number [28, 27]. The



phenomenon of the quark-number quenching due to quantum effects of the *massless* monopoles, has been further clarified in [29].

## 2.2. $USp(4)$ , $N_f = 4$

Another simple but very instructive example of nonabelian Argyres-Douglas vacua occurs in  $USp(4)$  theory with  $N_f = 4$  flavors. This system has been analyzed by Auzzi and Grena [31]. A characteristic feature of this model, which makes it really interesting, as compared to the  $SU(N)$  theories discussed above, is the fact that it possesses a nontrivial chiral symmetry,  $G_F = SO(8)$ <sup>5</sup>. The action of this theory is the standard  $\mathcal{N} = 2$  action with superpotential,

$$W = 1\wp\sqrt{2}Q_a^i\Phi_b^aQ_c^iJ^{bc} + m_{ij}\wp2Q_a^iQ_b^jJ^{ab}, \quad i, j = 1, 2, \dots, 2N_f, \quad (2.18)$$

where  $J = i\sigma_2 \otimes \mathbf{1}_N$  and

$$m = -i\sigma_2 \otimes \text{diag}(m_1, m_2, \dots, m_{N_f}). \quad (2.19)$$

In the  $m_i \rightarrow 0$  and  $\mu \rightarrow 0$  limit, the global symmetry is  $SO(2N_f) \times Z_{2N+2-N_f} \times SU_R(2)$ .

This theory ( $N_c = 2$ ,  $N_f = 4$ ) has seventeen vacua (for unequal  $m_i$  and for  $\mu \neq 0$ ). These degenerate into two ‘‘Chebyshev’’ vacua<sup>6</sup> and one ‘‘special’’ vacuum, in the limit,  $m_i \rightarrow 0$ . The Chebyshev vacua, which are the new SCFT, are the system of our interest here. Upon  $\mathcal{N} = 1$  perturbation

$$\Delta W = \mu \text{Tr} \Phi^2 \quad (2.20)$$

it can be shown that the special vacuum remains unconfined (free magnetic phase) while the two Chebyshev vacua are in confinement phase. To study the infrared properties of these vacua it is necessary to determine the light degrees of freedom.

This system has been carefully studied, by following the method used for the previous case [25], by Auzzi and Grena [31]. Their result is summarized in the following

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<sup>5</sup>In a nonsupersymmetric version of the model, the global symmetry would be  $SU(2N_f)$ , but the Yukawa interaction typical of  $\mathcal{N} = 2$  supersymmetry reduces the symmetry to  $SO(2N_f) \subset SU(2N_f)$ .

<sup>6</sup>These vacua are characterized by the point of the moduli  $\{\phi_i\}$ , which can be determined by use of some particular properties of the Chebyshev polynomials - trick first used by Douglas and Shenker [24].

table. They find that, as compared to the  $SU(3)$  SCFT considered above, there is one extra doublet in this system ( $C_1, C_2$  in Table. 2). The structure of the singularities (loci in the quantum moduli space where some dyon becomes massless) and the monodromies around each part of the singular loci, hence the charge determination of the Table. 2, have been double-checked independently by the present authors.

Particles	Charge
$M_1, M_2$	$(\pm 1, 1, 0, 0)^4$
$D_1, D_2$	$(\pm 2, -2, \pm 1, 0)$
$E_1, E_2$	$(0, 2, \pm 1, 0)$
$C_1, C_2$	$(\pm 2, 0, \pm 1, 0)$

Table 2: The charges of the massless doublets in one of the SCFT vacua.

Given the charges of the massless particles and given the symmetry breaking pattern,

$$SO(8) \rightarrow U(4), \quad (2.21)$$

(known from the analysis at large  $\mu$  [27]), we are forced to conclude that the monopole pair  $M, \tilde{M}$  condenses as

$$\langle M_a^i \tilde{M}_j^a \rangle = v \delta_j^i \neq 0, \quad a, b = 1, 2 \quad \text{and} \quad i, j = 1, \dots, 4. \quad (2.22)$$

Although the set of massless particles are rather similar to those found in the  $r = 2$  vacuum of the  $SU(3)$  theory, we do not expect “baryonlike” condensate (2.13) to form in this system. Other condensates such as

$$\langle C_a \tilde{D}^a \rangle, \quad \langle D_a \tilde{C}^a \rangle, \quad \langle C_a \tilde{C}^a \rangle, \quad (2.23)$$

etc., including the new doublet  $C$ , might well form, but would not modify the symmetry breaking pattern.

The difference in the massless spectrum and in the dynamics of this system, as compared to those in the  $SU(3)$  theory discussed in the previous section, can be attributed to the fact that these two SCFT’s belong to two different universality classes. See below for more about this point.

### 2.3. Light nonabelian monopoles in $USp(2N)$ theories

It has been shown [27] that *all* of the confining vacua of  $USp(2N)$  theories with  $m_i \rightarrow 0$ ,  $\mu \neq 0$ , with  $N_f \neq 0$  flavors, are perturbation of nontrivial superconformal vacua, as in the  $USp(4)$  example discussed above. (The same holds for  $SO(N)$  gauge theories as shown in [32]). The infrared physics is not described by a local effective Lagrangian and is difficult to analyze. However, the chiral symmetry breaking pattern

$$SO(2N_f) \rightarrow U(N_f) \tag{2.24}$$

is *known* from the behavior of the system at large  $\mu$  [27]. There is actually some intriguing similarity [27] between (2.24) and the symmetry breaking pattern in the real-world QCD, (6.2), so it is important to attempt to understand better the physics of this system.

It is possible that condensate analogous to (2.22) forms in general  $USp(2N)$  theory, but what are the global quantum numbers carried by the monopoles? Which kind of monopoles are present? How do they interact?

In the absence of the local effective action valid at low energies it is not an easy task to answer such questions. To work out the singularity structures and the monodromy matrices around each subsingular loci, as has been done in the simplest cases [25] and [31], seems to be out of question. Fortunately, one can vary certain parameters in the system and go to the regimes where physical interpretation is easier.

In particular, it is useful to vary the quark masses,  $m_i$ , although we are really interested in what happens in the  $m_i \rightarrow 0$  limit. First, let us choose nonvanishing but equal masses, *i.e.*,  $m_i = m \neq 0$ ,  $\forall i$ . It was found in [27] that each of our confining singularities of the  $USp(2N)$  theory (Chebyshev vacua) splits up into various singularities describing local  $SU(r) \times U(1)^{N-r}$  theories,  $r = 0, 1, \dots, \frac{N_f}{2}$ . Physics in each of these  $r$  vacua is similar as in the “ $r$ -vacua” in the  $SU(N)$  QCD, due to the universality of SCFT, as pointed out by Eguchi et. al. [22]. We know then, *assuming* that the light chiral multiplets present for  $m_i = m \neq 0$  survive in the  $m \rightarrow 0$  limit, that there are monopoles and dyons with various effective  $SU(r)$  charges, and in the fundamental representation of the global  $SU(N_f) \subset SO(2N_f)$  group.

We do know that in the SCFT limit ( $m \rightarrow 0$  limit), these monopoles all become massless simultaneously, but they are relatively non local, carrying mutually non zero Dirac units [13]. Local subset of fields, belonging to one of the  $r$  vacua, realize only

a subgroup  $SU(N_f)$  of the full symmetry group  $SO(2N_f)$ . On the other hand the Seiberg-Witten curve of this theory

$$xy^2 = \left[ x \prod_{a=1}^N (x - \phi_a^2) + 2\Lambda^{2N+2-N_f} m_1 \cdots m_{N_f} \right]^2 - 4\Lambda^{2(2N+2-N_f)} \prod_{i=1}^{N_f} (x + m_i^2) \quad (2.25)$$

has the correct flavor symmetry structure,  $SO(2N_f)$  in the  $m_i \rightarrow 0$  limit (see for instance [10]).

We have, apparently, an interesting new physical situation in which the global symmetry ( $SO(2N_f)$ ) of the system is realized by fields, mutually local subsets of which realize only a subgroup of the full symmetry group. This is perhaps not really surprising, once one accepts the fact that the system under study has no local low-energy effective action description.

As pointed out already, it is not easy to compute anything explicitly in these circumstances, but it is reasonable to assume that the symmetry breaking (Eq. (2.24)) is induced by the condensates of the monopoles  $M_a^i$  and  $\tilde{M}_j^a$ , with  $a = 1, 2, \dots, \left\lceil \frac{N_f}{2} \right\rceil$ ,

$$\langle M_a^i \tilde{M}_j^a \rangle = v \delta_j^i \neq 0, \quad i, j = 1, 2, \dots, N_f. \quad (2.26)$$

Note that these monopoles are the most strongly interacting fields, with  $SU\left(\left\lceil \frac{N_f}{2} \right\rceil\right)$  gauge interactions; other monopoles carrying  $SU(r)$  charges  $r < \left\lceil \frac{N_f}{2} \right\rceil$ , are more weakly coupled in the infrared <sup>7</sup>.

Eq. (2.26) also naturally generalizes the result of the  $USp(4)$  case, Eq. (2.22). Again, we do not expect baryonlike condensate (analogue of (2.16)) to form in this case, in contrast to what happens in  $SU(N)$  theories.

This is probably related to the absence of gauge-invariant baryonlike condensate (2.16) in the  $USp(2N)$  theory, which is instead present and crucial for  $SU(N)$  theories. This is a manifestation of the fact that the SCFT under consideration in the  $USp(2N)$  theory is in a different universality class from that of the  $r = \left\lceil \frac{N_f}{2} \right\rceil$  vacuum in  $SU(N)$  theory (as argued in [27]): they have different global symmetries, and light degrees of freedom and their interactions are distinct.

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<sup>7</sup>Actually in the limit  $m \rightarrow 0$ , the full gauge symmetry is recovered; it is not easy to see the effects of the  $USp(2N)$  interactions among the nonlocal sets of monopoles. The situation of the equal mass limit discussed in Section 2.1. was however different: there, the strong interactions among the monopoles arise only in the SCFT limit.

To understand better the behavior of the monopoles and to see the possible relevance of Goddard-Nuyts-Olive (GNO) monopoles [3], we looked into the perturbation (by masses  $m_i$ ) around the SCFT points, studied in [27], a little more carefully. For definiteness, let us consider the Chebyshev point of  $USp(2N)$  theory with  $N_f = 2N + 1$  and, instead of keeping nonvanishing equal masses and sending all of them to zero as done above, we first keep one mass much larger, and send others to 0 first, *i.e.*,  $\Lambda \gg m \gg m_i \rightarrow 0$ ,  $i = 2, 3, \dots, N_f$ . If  $m$  were large,  $m \gg \Lambda$ , the gauge group will be broken to  $USp(2N - 2) \times U(1)$  and a semiclassical reasoning will tell us that the system contains massive nonabelian monopoles, transforming in the dual gauge group  $SO(2N - 1)$ , see [1]-[6]. The number of the flavor ( $N_f = 2N + 1$ ) is chosen such that the “unbroken” group  $USp(2N - 2)$  is infrared free, so that it does not break itself further dynamically. We are particularly interested in knowing whether such monopoles survive quantum effects and become light, as  $m$  is reduced to smaller values, *i.e.*, less than  $\Lambda$ . The behavior of the theory in such a limit is encoded in the Seiberg-Witten curve, Eq. (2.25). The curve reduces at the mass scales much lower than  $m$  (where  $x \ll m$ , ) to the form (see Appendix A.):

$$x y^2 \simeq 4 m^2 \Lambda^2 \left[ \left( x \prod_{i=2}^N (x - \phi_i^2) - \prod_{i=2}^{N_f} m_i \right)^2 - \prod_{i=2}^{N_f} (x + m_i^2) \right], \quad (2.27)$$

which is nothing but the curve of  $USp(2N') = USp(2N - 2)$  theory with  $N'_f = N_f - 1 = 2N = 2N' + 2$ . By comparing this curve with the general form of the curve given in [10], one sees that it is a SCFT (in the limit  $m_i \rightarrow 0$ ) with infinitely strong coupling<sup>8</sup>. No hint of GNO monopoles, which would transform according to the dual gauge group  $SO(2N - 1)$ , is there.

If now another mass (*e.g.*,  $m_2 = m'$ ) is kept fixed and other masses ( $i = 3, 4, \dots$ ) are sent to zero first, the theory below the scale  $m, m'$  will behave as an  $USp(2N - 2)$  theory with one flavor less, an asymptotically free theory. But as  $g(\tau) = -1$  and the effective RG invariant scale is  $\Lambda' = g(\tau) m' \sim m'$ , the RG invariant scale of the theory is large. In other words the low energy theory is a strongly coupled system, and cannot be described by a weakly-coupled picture of monopoles.

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<sup>8</sup>(2.27) coincides with the curve given in [10] with  $g(\tau) = -1$ , where

$$g(\tau) \equiv \frac{\vartheta_2^4(\tau)}{\vartheta_3^4(\tau) + \vartheta_4^4(\tau)}, \quad \vartheta_2(\tau) = \sum_{n \in \mathbb{Z}} q^{(n+1/2)^2}; \quad \vartheta_3(\tau) = \sum_{n \in \mathbb{Z}} q^{n^2}; \quad \vartheta_4(\tau) = \sum_{n \in \mathbb{Z}} (-1)^n q^{n^2}.$$

An analogous result holds for even  $N_f$ , for instance,  $N_f = 2N$ : the theory in the regime  $\Lambda \gg m \gg m_i \rightarrow 0$ ,  $i = 2, 3, \dots, N_f$  is a strongly interacting system. The moral of the story is that there are no weakly-coupled monopoles acting as infrared degrees of freedom in the confining vacuum around the SCFT of the softly broken  $\mathcal{N} = 2$   $USp(2N)$  theory and, in particular, there are no hint of GNO monopoles becoming light by quantum effects in these theories.

Analogous conclusions apply also to the Chebyshev (SCFT) vacua of  $SO(N)$  theories.

### 3. Abelian versus nonabelian monopoles

As is well known, 't Hooft-Polyakov monopoles can acquire, upon quantization of matter fields, flavor quantum (quark) numbers through the fermion zero modes [30]. If such a monopole condenses, inducing confinement, the global symmetry is also broken dynamically, leading to a direct connection between confinement and dynamical symmetry breaking [8].

As was noted in [27], however, in certain vacua there are reasons to believe that abelian monopoles cannot possibly be the correct infrared degrees of freedom. For concreteness, let us first consider the case of  $\mathcal{N} = 2$  supersymmetric  $USp(2N)$  gauge theories with  $N_f$  fundamental matter multiplets, discussed in the preceding section, where the global symmetry is  $SO(2N_f)$ . According to the standard Jackiw-Rebbi analysis [30] the semiclassical 't Hooft-Polyakov monopoles of this theory are in an even or odd spinor representations of  $SO(2N_f)$ , each with multiplicity,  $2^{N_f-1}$ . The number of  $\mathcal{N} = 1$  vacua fits very nicely with it [27].

However, if the abelian monopoles were the light degrees of freedom the low-energy theory would have automatically a global, accidental  $SU(2^{N_f-1})$  symmetry, and would lead eventually to a huge number of Nambu-Goldstone bosons, not expected from the symmetry considerations alone.

Curiously, in the  $SU(2)$  theories studied by Seiberg and Witten [8], this does not cause a problem, apparently simply because  $2^{N_f-1}$  (for  $N_c = 2$ ,  $N_f$  is limited by  $N_f \leq 3$ ) is a small number! The light monopoles *are* found to be indeed 't Hooft-Polyakov abelian monopoles in the spinor representations of  $SO(4) \sim SU(2) \times SU(2)$  and of  $SO(6) \sim SU(4)$  for  $N_f = 2$  and  $N_f = 3$ , respectively [8]. Monopole condensation

leads to the symmetry breaking

$$SO(4) \rightarrow SU(2), \quad \text{or} \quad SO(6) \rightarrow SU(3), \quad (3.1)$$

with ensuing massless spectrum compatible with the standard Nambu-Goldstone theorem.

Once the rank of the gauge group is taken higher, one immediately faces a question: could it be possible that the effective low-energy theory possesses a global symmetry which is much larger than the true symmetry of the system, *i.e.*, a very large accidental symmetry? Is it possible that the low-energy spectrum contains many massless particles, expected neither from the Nambu-Goldstone theorem nor from any other principles (such as 't Hooft's anomaly matching condition)?

Actually, none of these embarrassing things occur. In the  $USp(2N)$  theory with matter, the potential abelian monopoles in the spinor representation of  $SO(2N_f)$ , actually are replaced by the nonabelian monopoles in the fundamentals of various  $SU(r)$  effective gauge groups, and transforming as  $\underline{N_f}$  or  $\underline{N_f}^*$  of the subgroup  $SU(N_f) \subset SO(2N_f)$ .

The replacement of the potential 't Hooft-Polyakov monopoles by nonabelian monopoles, is not a phenomenon restricted to the Argyres-Douglas vacua where monopoles are strongly coupled: it occurs also in an  $r$  vacuum of the  $SU(N)$  gauge theory, as was already noted in [27]. Physics of the  $r$  vacua in that theory is quite well understood, except the SCFT case  $r = \frac{N_f}{2}$  discussed in Section 2.1.: it is an effective  $SU(r)$  gauge theory with  $N_f$  dual quarks (nonabelian Goddard-Nuyts-Olive monopoles [3]), and confinement is induced by the condensation of these nonabelian monopoles. As long as the flavor quantum numbers are concerned, it looks as if abelian monopoles in the antisymmetric rank  $r$  tensor representation of the global  $SU(N_f)$  symmetry in these vacua had broken up [27] into “baryonic” components, as

$$M_{tHP}^{i_1 \dots i_r} \sim \epsilon^{a_1 \dots a_r} q_{a_1}^{i_1} q_{a_2}^{i_2} \dots q_{a_r}^{i_r}. \quad (3.2)$$

The nonabelian monopoles ( $q_a^i$ 's) carry  $\frac{1}{r}$  of the  $U(1)$  magnetic charge with respect to what is expected for a minimal *abelian* monopole. This is understood by the fact that the nontrivial homotopy group

$$\pi_1(U(r)) = \pi_1 \left( \frac{SU(r) \times U(1)}{\mathbb{Z}_r} \right) = \mathbb{Z} \quad (3.3)$$

is generated by the minimal loop involving the  $\mathbb{Z}_r$  element of  $SU(r)$ , that is, a  $\frac{1}{r}$  of the full circle in the  $U(1)$  gauge factor (see for instance the first of [6]).

Let us add a remark that the dual quarks in the Seiberg’s dual theory in  $SU(N)$  SQCD, can also be regarded as the “baryonic” components (with respect to the dual  $SU(\tilde{N})$  group) of the original baryonic composites  $B = \epsilon_{a_1 \dots a_N} Q^{a_1} Q^{a_2} \dots Q^{a_N}$  of the fundamental theory:

$$B \sim \epsilon_{b_1 \dots b_{\tilde{N}}} q^{b_1} q^{b_2} \dots q^{b_{\tilde{N}}}, \quad \tilde{N} = N_f - N. \quad (3.4)$$

In this case it is the constraint of the ’t Hooft’s anomaly matching (correct low-energy symmetry realization) that forces the system to choose Seiberg’s dual quark as the infrared degrees of freedom.

#### 4. Quantum vs classical $r$ -vacua of $SU(N)$ theories: Seiberg’s dual quarks as GNO monopoles

What is the physical meaning of Seiberg’s duality in  $\mathcal{N} = 1$  supersymmetric QCD? In spite of the observation made at the end of the preceding section, and in spite of overwhelming evidence for it [4], the nature of the Seiberg’s duals remains somewhat mysterious. Attempt to “understand” it starting from  $\mathcal{N} = 2$  supersymmetric QCD [28] was not entirely successful.

A clue to the meaning of the Seiberg’s dual quarks comes from the exact relation between the quantum and classical  $r$ -vacua appearing in the  $\mathcal{N} = 2$  supersymmetric  $SU(N)$  QCD [34]. For definiteness and for concreteness let us restrict ourselves to the cases of  $SU(N)$  gauge groups in this section.

In Carlino et. al. [27] the matching of the *total number* of semiclassical (at large  $\mu$  and  $m_i$ ) and quantum mechanical (at small  $\mu$  and  $m_i$ )  $\mathcal{N} = 1$  vacua has been worked out carefully. Although not stated very explicitly there, the counting and matching work out for the vacua having a definite symmetry, actually: we are able to “follow” the flow of each vacuum from the semiclassical region to fully quantum mechanical regime<sup>9</sup>. First consider the case  $N_f \leq N$ . It is seen from (2.3) and (2.4) that there

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<sup>9</sup>As in Carlino et. al. [27], we keep nonvanishing and generic masses  $\mu$  and  $m_i$  so that only discrete set of  $\mathcal{N} = 1$  vacua remain.



are total of

$$\sum_{r=0}^{N_f} (N-r) \binom{N_f}{r} = (2N - N_f) 2^{N_f-1} \quad (4.1)$$

vacua, where  $r$  is the number of the massless flavors in the semiclassical theory. In fact, the index  $r$  characterizes both the local gauge symmetry  $(SU(r) \times U(1)^{N-r})$  and the flavor symmetry  $(U(r) \times U(N_f - r))$  of each vacuum.

In the counting of classical vacua (4.1) the integer  $r$  runs from 0 to  $N_f$ . The number of the vacua with global  $U(r_0) \times U(N_f - r_0)$  symmetry ( $r_0 < \frac{N_f}{2}$ ) is given by the sum of those with  $r = r_0$  and  $r = N_f - r_0$ ,

$$(N - r_0) \binom{N_f}{r_0} + \{N - (N_f - r_0)\} \binom{N_f}{N_f - r_0} = (2N - N_f) \binom{N_f}{r_0}, \quad (4.2)$$

which precisely matches the number of the quantum  $r$  vacua with  $r = r_0$ . Note that in the quantum case  $2N - N_f$  is the multiplicity due to the discrete  $\mathbb{Z}_{2N-N_f}$  symmetry present in the  $m_i \rightarrow 0$  limit;  $r_0$  runs only up to  $\frac{N_f}{2}$ . This last fact can be understood on the basis of the renormalization group argument [33]. Summarizing, the classical and quantum  $r$  vacua correspond as

$$\{r_0, N_f - r_0\} \rightarrow r_0, \quad r_0 \leq \frac{N_f}{2}. \quad (4.3)$$

This analysis is useful, not so much as a further consistency check but because it tells us something nontrivial about physics. Semiclassically the  $SU(r)$  gauge theory with  $r = r_0 < \frac{N_f}{2}$  is infrared free<sup>10</sup>, so it is natural that it survives in the infrared, even though the quarks “become” monopoles due to the rearrangement of singularities (phenomenon of “isomonodromy” discussed in [8, 21, 35]).

The theory with classical  $SU(r)$  gauge symmetry with  $r = N_f - r_0 > \frac{N_f}{2}$ , instead, is asymptotically free and becomes strongly coupled in the infrared. The vacuum counting and matching above teach us that *this theory is replaced in the infrared by an  $SU(N_f - r) = SU(r_0)$  theory with  $N_f$  flavors of nonabelian monopoles*. This is nothing but the Seiberg’s duality [4].

The important point is that the monopoles in these vacua can be identified [33] at the same time also as the nonabelian Goddard-Nuyts-Olive monopoles [3] associated with the partial gauge symmetry breaking

$$SU(N) \rightarrow SU(r_0) \times U(1)^{N-r_0}, \quad r_0 < \frac{N_f}{2} \leq N, \quad (4.4)$$

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<sup>10</sup>We recall that the beta function of the  $\mathcal{N} = 2$  SQCD is proportional to  $N_f - 2N_c$ .

since the fundamental theory contains the adjoint scalar  $\Phi$ . This provides a precious bridge between the concept of semiclassical nonabelian monopoles and that of Seiberg's dual quarks.

And this hints at the interpretation of Seiberg's dual quarks as the GNO monopoles also in the standard,  $\mathcal{N} = 1$  supersymmetric QCD. Let us recall that for  $N + 1 < N_f \leq 3N$  the concept of magnetic quarks make sense, the dual theory is a  $SU(\tilde{N}) = SU(N_f - N)$  theory, while for  $\frac{3N}{2} \leq N_f$  the concept of the quark field make sense as the infrared degrees of freedom. In the conformal window,

$$\frac{3N}{2} \leq N_f \leq 3N \quad (4.5)$$

either description can be used, and the theory flows into an infrared fixed-point theory (superconformal theory). Now, when  $N_f < 2N$  the original  $SU(N)$  theory is more strongly coupled in the infrared: it is natural to consider it as the “fundamental” theory and  $SU(\tilde{N})$  as its dual ( $\tilde{N} < N$ ); for  $N_f > 2N$  the  $SU(\tilde{N})$  theory might be considered as fundamental, the quarks are then GNO “monopoles”. Of course, the  $\mathcal{N} = 1$  SQCD does not have an elementary scalar field in the adjoint representation, and there are no classical soliton monopoles. Nevertheless the theory seems to produce dynamically magnetic soliton-like monopoles, which act as the correct infrared degrees of freedom. And this, in turn, seems to suggest that an analogous phenomenon is possible in the ordinary QCD.

The correspondence between classical and quantum vacua are subtler in the case,  $N_f > N$ . One must distinguish the cases with  $r \geq N_f - N$  and those with  $r < N_f - N$ . For the vacua  $r \geq N_f - N$  the discussion analogous to the one done for  $N_f \leq N$  holds, there are a pair of classical vacua  $(r_0, N_f - r_0)$  in which the symmetry of the system is  $U(r_0) \times U(N_f - r_0)$ . Their sum give

$$\sum_{r_0=N_f-N}^N (N - r_0) \binom{N_f}{r_0} = \sum_{r_0=N_f-N}^{N_f/2} (2N - N_f) \binom{N_f}{r_0}. \quad (4.6)$$

The vacua with smaller  $r = r_0$ ,  $r_0 < N_f - N$  appear alone: their total number is

$$\sum_{r_0=0}^{N_f-N-1} (N - r_0) \binom{N_f}{r_0} \quad (4.7)$$

Quantum mechanically, we know that there are two kinds of vacua, those in the

confinement phase appear

$$\mathcal{N}_1 = \sum_{r_0=0}^{N_f/2} (2N - N_f) \binom{N_f}{r_0} \quad (4.8)$$

times, while the number of the vacua in a free magnetic phase (no confinement) is

$$\mathcal{N}_2 = \sum_{r_0=0}^{N_f-N-1} (N_f - N - r_0) \binom{N_f}{r_0}. \quad (4.9)$$

In order to find the matching, split the sum in (4.6) as

$$\sum_{r_0=N_f-N}^{N_f/2} (2N - N_f) \binom{N_f}{r_0} = \sum_{r_0=0}^{N_f/2} (2N - N_f) \binom{N_f}{r_0} - \sum_{r_0=0}^{N_f-N-1} (2N - N_f) \binom{N_f}{r_0}. \quad (4.10)$$

The first term is equal to  $\mathcal{N}_1$ , while the sum of the second term with (4.7) is precisely  $\mathcal{N}_2$ .

## 5. GNO monopoles in $USp(2N)$ , $SO(2N+1)$ theories

We have presented in Section 2.3. some evidence against the appearance of light GNO monopoles associated with partial  $USp(2N)$  gauge group breaking in theories with  $\mathcal{N} = 2$  supersymmetry. Actually there is a simple reason why light monopoles of GNO type cannot appear as light degrees of freedom, at least in the context of  $\mathcal{N} = 2$  supersymmetric theories, with  $USp(2N)$  or  $SO(N)$  groups. Suppose that the gauge symmetry is partially broken, for instance, as

$$USp(2N) \rightarrow USp(2N-2) \times U(1). \quad (5.1)$$

The semiclassical monopoles representing the homotopy group

$$\pi_2 \left( \frac{USp(2N)}{USp(2N-2) \times U(1)} \right) \sim \pi_1(USp(2N-2) \times U(1)) \quad (5.2)$$

would transform as in the fundamental representation of  $SO(2N-1) \times U(1)$ , dual of  $USp(2N-2) \times U(1)$ . Now in the presence of  $N_f$  massless flavors (which is needed to prevent  $USp(2N-2)$  from breaking itself dynamically to abelian subgroups), the fundamental theory has a global  $SO(2N_f)$  symmetry, as already mentioned. Now if the GNO monopoles appeared in the low energies, the low-energy effective theory

would have an  $USp(2N_f)$  symmetry, instead of the correct  $SO(2N_f)$  symmetry<sup>11</sup>. The global symmetry appears to prevent the GNO monopoles from becoming light in these circumstances, and indeed this does not occur!

An analogous situation presents itself in  $SO(2N + 1)$  gauge theories with  $N_f$  flavors in vector representation, where the global symmetry is  $USp(2N_f)$ . The GNO monopoles associated with partial breaking, *e.g.*,  $SO(2N + 1) \rightarrow SO(2N - 1) \times U(1)$ , cannot become light, as it would imply a wrong symmetry in the low energies, and indeed, they do not appear.

## 6. Discussion

In this paper we have discussed various aspects of nonabelian Argyres-Douglas vacua which occur frequently in supersymmetric theories. The main observation made here is that, besides the renormalization group features already emphasized in [33, 6], there are other constraints, mostly related to the realization of the global symmetries, which severely restrict which types of monopoles can appear as the light degrees of freedom. We have analysed different classes of  $SU(N)$  and  $USp(2N)$  theories (mentioning also briefly results on  $SO(N)$  theories) and found a number of interesting phenomena. Although the fact that we are able to analyse and get information does depend on the presence of supersymmetry, we believe that the phenomena themselves are of much more general nature.

The most interesting, and quite general, phenomenon is the replacement of abelian monopoles by nonabelian monopoles as the infrared degrees of freedom. We found various cases in which the system is forced to choose the latter, in order to realize appropriately the global symmetry of the underlying theory, *e.g.*, not to have too large an accidental symmetry.

Secondly, in a wide class of nonabelian Argyres-Douglas vacua studied here, the confinement (which requires a partial supersymmetry breaking) is induced by the condensation of strongly-interacting nonabelian monopole composites. These monopoles carry the flavor charges, and the pattern of global symmetry breaking is determined by the type of *composites* of nonabelian monopoles which condense, such as (2.13), (2.16), (2.22) and (2.26).

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<sup>11</sup>We thank H. Murayama on the discussion on this point.

Thirdly - this seems to be the most important feature valid in wide classes of supersymmetric models - the most interesting confining vacua with nonabelian symmetry, namely, confinement neither accompanied by dynamical abelianization nor by infrared-free dual gauge interactions, occurs near (*i.e.*, perturbation of) nontrivial conformal vacua.

What can one learn from these studies for QCD? Let us speculate on what might be possibly happening, drawing analogies from some of the phenomena found here. In the standard QCD, the lattice calculations tell us that the chiral symmetry restoration and deconfinement transitions take place at the same temperature, suggesting a close dynamical connection between the two phenomena. There are no hint of dynamical abelianization, so that if the ground state of QCD is a kind of dual superconductor, it must be of a nonabelian variety [17]. The  $SU(3)$  color group is not broken dynamically to  $U(1) \times U(1)$ ; let us assume that the dual theory is instead  $SU(2) \times U(1)$ . The monopoles  $M_\alpha^i$  and  $\tilde{M}_j^\alpha$  would carry the dual color index  $\alpha = 1, 2$  and the flavor indices  $i$  (of  $SU_L(N_f)$ ) and  $j$  (of  $SU_R(N_f)$ ), respectively. As there are no phenomenological evidences that the long-distance hadronic physics is governed by weakly-coupled magnetic monopoles, we must assume that they interact strongly.

We speculate that the condensate

$$\langle M_\alpha^i \tilde{M}_j^\alpha \rangle = v \delta_j^i \neq 0, \quad i, j = 1, 2, \dots, N_f, \quad v \sim O(\Lambda_{QCD}^2) \quad (6.1)$$

is formed, due to the strong dual gauge interactions, inducing confinement, and at the same time causing the symmetry breaking

$$SU_L(N_f) \times SU_R(N_f) \times U_A(1) \times U_V(1) \rightarrow SU_V(N_f) \times U_V(1). \quad (6.2)$$

One might wonder why the condensing entity cannot simply be a 't Hooft-Polyakov (or 't Hooft-Mandelstam) monopole, dressed with flavor quantum numbers of bifermion composite,  $\bar{\psi}_R \psi_L$ . It would however be inconsistent to assume that such abelian monopoles are the dominant effective degrees of freedom. A  $U(1)$  theory with  $N_f^2$  monopoles would have a global symmetry,  $SU(N_f^2)$ , and the disaster of “too-many-Nambu-Goldstone bosons” discussed in Section 3. would ensue. As the supersymmetric systems discussed there, the standard  $SU(3)$  QCD would probably avoid such an awkward situation by producing nonabelian monopoles (even at the price of having them necessarily strongly interacting) as the effective degrees of freedom.

The condensate (6.1) carries the same flavor quantum numbers as the standard quark bilinear condensate,  $\langle \bar{\psi}_{Rj} \psi_L^i \rangle$ . It is quite possible that these two types of

condensates are actually dynamically related, either by a nonabelian Jackiw-Rebbi mechanism (through the fermion zero-modes), or by a generalization of the Rubakov effect [36].

A somewhat related idea, that the vacuum of QCD is close to a nontrivial infrared fixed-point theory and that such a conformal invariance is achieved by the collaboration of relatively nonlocal fields, has been discussed recently by C.R. Das et. al. [37].

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## References

- [1] E. Lubkin, **Ann. Phys.** **23** (1963) 233; E. Corrigan, D.I. Olive, D.B. Fairlie and J. Nuyts, **Nucl. Phys.** **B106** (1976) 475.
- [2] C. Montonen and D. Olive, **Phys. Lett.** **72 B** (1977) 117.
- [3] P. Goddard, J. Nuyts and D. Olive, **Nucl. Phys.** **B125** (1977) 1; F.A. Bais, **Phys. Rev.** **D18** (1978) 1206; E. J. Weinberg, **Nucl. Phys.** **B167** (1980) 500; **Nucl. Phys.** **B203** (1982) 445; N. Dorey, T.J. Hollowood and M.A.C. Kneipp, [arXiv:hep-th/9512116] (1995); N. Dorey, C. Fraser, T.J. Hollowood and M.A.C. Kneipp, **Phys. Lett.** **B383** (1996) 422 [arXiv:hep-th/9605069]; C. J. Houghton, P. M. Sutcliffe, **J. Math. Phys.** **38** (1997) 5576 [arXiv:hep-th/9708006], B.J. Schroers and F.A. Bais, **Nucl. Phys.** **B535** (1998) 197 [arXiv:hep-th/9805163].
- [4] N. Seiberg, **Nucl. Phys.** **B435** (1995) 129 [arXiv:hep-th/9411149].
- [5] D. Kutasov, **Phys. Lett. B** **351** (1995) 230 [arXiv:hep-th/9503086]; D. Kutasov and A. Schwimmer, **Phys. Lett. B** **354** (1995) 315 [arXiv:hep-th/9505004]; D. Kutasov, A. Schwimmer and N. Seiberg, **Nucl. Phys. B** **459** (1996) 455 [arXiv:hep-th/9510222].

- [6] R. Auzzi, S. Bolognesi, J. Evslin, K. Konishi and H. Murayama, **Nucl. Phys. B** **701** (2004) 207 [arXiv:hep-th/0405070], K. Konishi, in “Continuous Advances in QCD 2004”, Minneapolis, 2004, Edit. T. Gherghetta, World Scientific (2004), [arXiv:hep-th/0407272].
- [7] N. Seiberg and E. Witten, **Nucl. Phys. B****426** (1994) 19 [arXiv:hep-th/9407087] (Erratum *ibid.* **B430** (1994) 485).
- [8] N. Seiberg and E. Witten, **Nucl. Phys. B****431** (1994) 484 [arXiv:hep-th/9408099].
- [9] P.C. Argyres and A.E. Faraggi, **Phys. Rev. Lett** **74** (1995) 3931 [arXiv:hep-th/9411047]; A. Klemm, W. Lerche, S. Theisen and S. Yankielowicz, **Phys. Lett. B****344** (1995) 169 [arXiv:hep-th/9411048]; A. Klemm, W. Lerche and S. Theisen, **Int. J. Mod. Phys. A****11** (1996) 1929 [arXiv:hep-th/9505150]; A. Hanany and Y. Oz, **Nucl. Phys. B****452** (1995) 283 [arXiv:hep-th/9505075]; P. C. Argyres, M. R. Plesser and A. D. Shapere, **Phys. Rev. Lett.** **75** (1995) 1699 [arXiv:hep-th/9505100]; A. Hanany, **Nucl. Phys. B****466** (1996) 85 [arXiv:hep-th/9509176].
- [10] P. C. Argyres and A. D. Shapere, **Nucl. Phys. B****461** (1996) 437 [arXiv:hep-th/9509175].
- [11] R. Dijkgraaf and C. Vafa, **Nucl. Phys. B** **644** (2002) 3 [arXiv:hep-th/0206255]; R. Dijkgraaf and C. Vafa, **Nucl. Phys. B** **644** (2002) 21 [arXiv:hep-th/0207106]; R. Dijkgraaf and C. Vafa, arXiv:hep-th/0208048.
- [12] F. Cachazo, M. R. Douglas, N. Seiberg and E. Witten, **JHEP** **0212** (2002) 071 [arXiv:hep-th/0211170]; F. Cachazo, N. Seiberg and E. Witten, **JHEP** **0302** (2003) 042 [arXiv:hep-th/0301006], **JHEP** **0304** (2003) 018 [arXiv:hep-th/0303207].
- [13] G. 't Hooft, **Nucl. Phys. B****190** (1981) 455; S. Mandelstam, **Phys. Lett.** **53B** (1975) 476, **Phys. Rep.** **23C** (1976) 245.
- [14] A. Yung, “What do we learn about confinement from Seiberg-Witten theory”, Talk given at the 3rd Moscow School of Physics and 28th ITEP Winter School of Physics, Moscow, Russia, 22 Feb - 3 Mar 2000, [arXiv: hep-th/0005088].

- [15] M. Strassler, **Prog. Theor. Phys. Suppl.** **131** (1998) 439 [hep-lat/9803009].
- [16] A. Hanany and D. Tong, **JHEP** **0307:037** (2003) [hep-th/0306150]; **JHEP** **0404:066** (2004) [hep-th/0403158]
- [17] R. Auzzi, S. Bolognesi, J. Evslin, K. Konishi and A. Yung, **Nucl. Phys.** **B673** (2003) 187 [hep-th/0307287]; R. Auzzi, S. Bolognesi, J. Evslin and K. Konishi, **Nucl. Phys.** **B686** (2004) 119 [hep-th/0312233].
- [18] A. Gorsky, M. Shifman and A. Yung, **Phys. Rev.** **D71** (2005) 045010 [arXiv:hep-th/0412082]; R. Auzzi, M. Shifman and A. Yung, **Phys. Rev.** **D72** (2005) 025002 [arXiv:hep-th/0504148]; N. Sakai and D. Tong, **JHEP** **0503** (2005) 019 [arXiv:hep-th/0501207]; M. Eto, Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, “Moduli Space of Non-Abelian Vortices”, [arXiv:hep-th/0511088].
- [19] P. C. Argyres and M. R. Douglas, **Nucl. Phys.** **B448** (1995) 93 [arXiv:hep-th/9505062].
- [20] P. C. Argyres, M. R. Plesser, N. Seiberg and E. Witten, **Nucl. Phys.** **B461** (1996) 71 [arXiv:hep-th/9511154], for a recent development, see P.C. Argyres, M. Crescimanno, A.D. Shapere and J.R. Wittig, “Classification of N=2 superconformal field theories with two-dimensional Coulomb branches,” [arXiv:hep-th/0504070].
- [21] A. Bilal and F. Ferrari, **Nucl. Phys.** **B516** (1998) 175 [arXiv:hep-th/9706145].
- [22] T. Eguchi, K. Hori, K. Ito and S.-K. Yang, **Nucl. Phys.** **B471** (1996) 430 [arXiv:hep-th/9603002].
- [23] T. Kubota and N. Yokoi, **Prog. Theor. Phys.** **100** (1998) 423 [arXiv:hep-th/9712054].
- [24] M. R. Douglas and S. H. Shenker, **Nucl. Phys.** **B447** (1995) 271 [arXiv:hep-th/9503163].
- [25] R. Auzzi, R. Grena and K. Konishi, **Nucl. Phys.** **B653** (2003) 204 [arXiv:hep-th/0211282].
- [26] A. Gorsky, A. Vainshtein and A. Yung, **Nucl. Phys.** **B584** (2000) 197 [arXiv:hep-th/0004087].



- [27] G. Carlino, K. Konishi and H. Murayama, **JHEP** **0002** (2000) 004 [arXiv:hep-th/0001036]; **Nucl. Phys. B****590** (2000) 37 [arXiv:hep-th/0005076].
- [28] P. C. Argyres, M. R. Plesser and N. Seiberg, **Nucl. Phys. B****471** (1996) 159 [arXiv:hep-th/9603042]; P.C. Argyres, M.R. Plesser, and A.D. Shapere, **Nucl. Phys. B****483** (1997) 172 [arXiv:hep-th/9608129]; K. Hori, H. Ooguri and Y. Oz, **Adv. Theor. Math. Phys.** **1** (1998) 1 [arXiv:hep-th/9706082].
- [29] G. Carlino, K. Konishi and H. Terao, **JHEP** **9804** (1998) 003 [arXiv:hep-th/9801027].
- [30] R. Jackiw and C. Rebbi, **Phys. Rev. D****13** (1976) 3398.
- [31] R. Auzzi and R. Grena, “Superconformal Vacua in N=2  $USp(4)$  Gauge Theories,” [arXiv:hep-th/0402213], R. Auzzi, PhD Thesis at Scuola Normale Superiore, Pisa (2004), R. Grena, PhD Thesis at University of Pisa, Pisa (2005).
- [32] G. Carlino, K. Konishi, Prem Kumar and H. Murayama, **Nucl. Phys. B****608** (2001) 51 [arXiv:hep-th/0104064].
- [33] S. Bolognesi and K. Konishi, **Nucl. Phys. B****645** (2002) 337 [arXiv:hep-th/0207161].
- [34] S. Bolognesi, K. Konishi and G. Marmorini, **Nucl. Phys. B****718** (2005) 134 [arXiv:hep-th/0502004].
- [35] A. Cappelli, P. Valtancoli and L. Vergnano, **Nucl. Phys. B****524** (1998) 469 [arXiv:hep-th/9710248].
- [36] V.A. Rubakov, **Nucl. Phys. B****203** (1982) 311.
- [37] C.R. Das, L.V. Laperashvili and H.B. Nielsen, “Generalized dual symmetry of nonabelian theories, monopoles and dyons,” [arXiv:hep-th/0510392].

## Appendix A: Light monopoles at the Chebyshev point of $USp(2N)$ theory

The curve of the theory is given by

$$xy^2 = \left[ x \prod_{a=1}^N (x - \phi_a^2) + 2\Lambda^{2N+2-N_f} m_1 \cdots m_{N_f} \right]^2 - 4\Lambda^{2(2N+2-N_f)} \prod_{i=1}^{N_f} (x + m_i^2). \quad (\text{A.1})$$

For definiteness first consider the case of odd number of flavors, with  $N_f = 2N + 1$ . The Chebyshev solution corresponds to  $\phi_a = 0, \forall a$ :

$$xy^2 = [x^{N+1}]^2 - 4\Lambda^2 x^{2N+1} = x^{2N+1}(x - 4\Lambda^2). \quad (\text{A.2})$$

The zero at  $x = 0$  is of degree  $2N$ , and there is another isolated zero at  $x = 4\Lambda^2$ . There is also a branch point at  $x = \infty$ .

Under the perturbation by generic quark masses, the problem is to find  $\{\phi\}$ 's such that the curve factorizes with maximal number of double factors as

$$xy^2 = x(x - 4\Lambda^2 - \beta) \prod_{a=1}^N (x - \alpha_a)^2, \quad (\text{A.3})$$

and to find out in how many ways this can be done. This problem has been solved in Section 9.2. of [27]; the answer is

$$s_k(\alpha) = s_{2k}(m), \quad s_{k+1}(\phi^2) = -2\Lambda(-1)^N s_{2k+1}(m), \quad k = 0, 1, 2, \dots, \quad (\text{A.4})$$

where the symmetric polynomials  $s_j(\rho)$  are defined by:

$$\prod_{i=1}^N (z - \rho_i) = \sum_{k=0}^N (-1)^k s_k(\rho) z^{N-k}, \quad (\text{A.5})$$

that is,  $s_0(\rho) = 1$ ,  $s_1(\rho) = \sum_{i=1}^N \rho_i$ ,  $s_2(\rho) = \sum_{i<j} \rho_i \rho_j$ , etc.

We now consider a particular set of masses,  $m \gg m_i$ ,  $i = 2, 3, \dots, N_f$ . From Eq. (A.4) it follows that

$$s_1(\phi^2) = \sum_{i=1}^N \phi_i^2 = 2\Lambda \sum m_i \simeq 2\Lambda m;$$

$$s_2(\phi^2) = \sum_{i<j} \phi_i^2 \phi_j^2 \simeq \phi_1^2 s_1'(\phi^2) = 2\Lambda s_3(m_i) \simeq 2\Lambda m s_2'(m_i);$$

$$s_3(\phi^2) \simeq \phi_1^2 s'_2(\phi^2) = 2 \Lambda m s'_4(m_i);$$

etc., and in general

$$s'_k(\phi^2) \simeq s'_{2k}(m_i).$$

As for  $\alpha$ , one finds

$$s_1(\alpha) \simeq m s'_1(m_i); \quad s_2(\alpha) \simeq m s'_3(m_i); \quad s_1(\alpha) \simeq m s'_5(m_i),$$

and so on, and in general

$$\alpha_1 \simeq m s'_1(m_i), \quad s'_k(\alpha) \simeq \frac{s'_{2k+1}(m_i)}{s'_1(m_i)}. \quad (\text{A.6})$$

The prime above means  $\phi_1^2$ ,  $\alpha_1$  and  $m_1$  do not appear. The above results suggest that

$$\phi_1^2 \sim 2 m \Lambda, \quad \phi_i^2 = O(m_j^2), \quad i = 2, 3, \dots \quad (\text{A.7})$$

The curve then looks like (where  $x \ll m$ )

$$x y^2 \simeq 4 m^2 \Lambda^2 \left[ \left( x \prod_{a=2}^N (x - \phi_a^2) - \prod_{i=2}^{N_f} m_i \right)^2 - \prod_{i=2}^{N_f} (x + m_i^2) \right], \quad (\text{A.8})$$

which is nothing but the curve of  $USp(2N') = USp(2N - 2)$  theory with  $N'_f = N_f - 1 = 2N = 2N' + 2$ . It is a SCFT with  $g(\tau) = -1$  (infinitely strong coupling) [10], where

$$g(\tau) = \frac{\vartheta_2^4}{\vartheta_3^4 + \vartheta_4^4}. \quad (\text{A.9})$$

If now another mass (*e.g.*,  $m_2 = m'$ ) is kept fixed and other masses are sent to zero, the theory below the scale  $m, m'$  will behave as an  $USp(2N - 2)$  theory with one flavor less, an asymptotically free theory. But as  $g(\tau) = -1$  and the effective RG invariant scale is  $\Lambda' = g(\tau) m' \sim m'$ , the RG invariant scale of the theory is large. In other words the low energy theory cannot be described by a weakly coupled theory.

The Chebyshev point of  $USp(2N)$  theory with even number of flavors,  $N_f = 2N$ , with one mass kept finite,  $\Lambda \gg m \gg m_i \rightarrow 0$ ,  $i = 2, 3, \dots, N_f$ , can be treated in a similar manner. We first note that at  $\Lambda \gg m_i$ ,  $\forall i$ , the curve effectively reduces [27] to

$$x y^2 \simeq (2 \Lambda^2)^2 \left[ x \prod_{a=1}^{N-1} (x - \phi_a^2) - m_1 \cdots m_{N_f} \right]^2 - \prod_{i=1}^{N_f} (x + m_i^2), \quad (\text{A.10})$$

which is an SCFT with  $g = -1$ , as

$$N' = N - 1, \quad N_f = 2N = 2N' + 2. \quad (\text{A.11})$$

Again, by choosing one of the bare masses to be large as compared to the others,  $m_1 \gg m_i$ ,  $i = 2, 3, \dots$ , one gets effectively an asymptotically free theory with

$$N' = N - 1, \quad N'_f = 2N' + 1, \quad (\text{A.12})$$

and with the effective RG invariant scale,

$$\Lambda' = m_1, \quad (\text{A.13})$$

which is large at the mass scale  $m_i \ll m_1$ . The theory is strongly coupled.

Analogous conclusions apply to the Chebyshev (SCFT) vacua of  $SO(N)$  theories with massless matter, as can be shown by using the results of [32].